**Example 1:** If A, B, and C are sets, then A − (B ∪ C ) = (A − B) ∩ (A − C ).

**Proof.**

Suppose A, B and C are sets, Then

A − (B ∪ C ) ≡ {x | x ∈ A − (B ∪ C )}

≡ {x | (x ∈ A) ∧ (x (B ∪ C ))}

≡ {x | (x ∈ A) ∧ (x ∈ (B ∪ C )’)}

≡ {x | (x ∈ A) ∧ (x ∈ (B’ ∩ C’))}

≡ {x | (x ∈ A) ∧ ((x ∈ B’) ∧ (x ∈ C’))}

≡ {x | (x ∈ A ∧ x ∈ B’) ∧ (x ∈ A ∧ x ∈ C’)}

≡ {x | (x ∈ A ∧ x B) ∧ (x ∈ A ∧ x C)}

≡ {x | x ∈ (A − B) ∧ x ∈ (A − C)}

≡ {x | x ∈ (A − B) ∩ (A − C)}

≡ (A − B) ∩ (A − C).

**Example 1:** Suppose A, B, and C are sets.

If B ⊆ C, then A × B ⊆ A × C

**Proof.**

Let sets A, B, and C be given with B ⊆ C. Then

A × B = {(a, b) : a ∈ A ∧ b ∈ B}

Let (x, y ) ∈ A × B. Then x ∈ A and y ∈ B.

Since B ⊆ C , we know y ∈ C ,

so it must be that (x, y ) ∈ A × C .

Thus A × B ⊆ A × C .

**Example 2:** Given sets A, B, and C,

prove A × (B ∩ C) = (A × B) ∩ (A × C).

**Proof.** Just observe the following sequence of equalities.

A × (B ∩ C)

≡ {(x, y) | (x ∈ A) ∧ (y ∈ B ∩ C)} (def. of ×)

≡ {(x, y) | (x ∈ A) ∧ (y ∈ B) ∧ (y ∈ C)} (def. of ∩)

≡ {(x, y) | (x ∈ A) ∧ (x ∈ A) ∧ (y ∈ B) ∧ (y ∈ C)} (P = P ∧ P)

≡ {(x, y) | ((x ∈ A) ∧ (y ∈ B)) ∧ ((x ∈ A) ∧ (y ∈ C))} (rearrange)

≡ {(x, y) | (x ∈ A) ∧ (y ∈ B)} ∩ {(x, y) : (x ∈ A) ∧ (y ∈ C)} (def. of ∩)

≡ (A × B) ∩ (A × C) (def. of ×)

**Example 4.** Let A and B be sets. Then prove that,

A ⊕ B = (A ∪ B) \ (A ∩ B).

**Proof.**

A ⊕ B = (A \ B) ∪ (B \ A)

= (A ∩ Bc) ∪ (B ∩ Ac)

= ((A ∩ Bc) ∪ B) ∩ ((A ∩ Bc) ∪ Ac)

= [(A ∪ B) ∩ (Bc ∪ B)] ∩ [(A ∪ Ac) ∩ (Bc ∪ Ac)]

= [(A ∪ B) ∩ U] ∩ [U ∩ (Bc ∪ Ac)]

= (A ∪ B) ∩ (Bc ∪ Ac)

= (A ∪ B) ∩ (A ∩ B)c

= (A ∪ B) \ (A ∩ B)